A NEW METHOD OF DETERMINING THE THERMAL ACTIVITY OF NON-METALS

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A method is described for determining the thermal activity of nonmetallic materials from the magnitude of the control current flowing in the measuring circuit. This method does not require the use of thermocouples and automatic potentiometric recorders.

To determine the thermal activity coefficient $b = \lambda/\sqrt{a}$, it is proposed to measure the electrical energy generated in a plane heater located at the junction of the two semi-infinite rods, necessary to maintain constant heater temperature. *

A bridge circuit (figure) with negative feedback [1] may be used as the measuring circuit. One of the bridge arms contains a metal wire** heater made of a material (platinum, tungsten) with high electrical resistivity.

The bridge is balanced at an assigned heater temperature established beforehand by adjustment of the variable resistor R_l in the arm adjacent to the heater. When the circuit is closed, the heater is cold, the bridge imbalance is a maximum, and so the amplifier acts on the bridge supply to develop maximum power in the heater. As the heater warms up, the bridge imbalance decreases, tending to a very small value determined by the amplification factor of the measuring system.

To obtain quantitative relations and calculation formulas, we turn to a mathematical description of the unsteady processes in the measuring circuit and of the interaction of the heater with the test object. For this purpose we write the heat balance equation

$$C_h \frac{dT_h}{d\tau} - 2\lambda \left[\frac{\partial T(x,\tau)}{\partial x}\right]_{x=0} F = I_h^2 R_h. \qquad (1)$$

We express the heater resistance R_h in terms of its temperature

$$R_h \approx R_{h0}\,[1 + \beta_{h0}(\vartheta_h - \vartheta_{h0})], \qquad (2)$$

where R_{h0} is the heater resistance at temperature T_{h0}, $\vartheta_h = T_h - \Theta$, $\vartheta_{h0} = T_{h0} - \Theta$, β_{h0} is the temperature coefficient of heater resistance evaluated at temperature T_{h0} and F is the surface area of the heater.

*Methods using measuring probes have been proposed by Chudnovskii [3].

**It would be better to use a film heater made of semi-conductor material with a temperature coefficient of resistance an order larger than that of metals.

To obtain the derivative $\partial T/\partial x$, we solve the heat conduction equation for an infinite rod with a heat flux at its end such that its temperature remains unchanged. Mathematically the problem is

$$\frac{\partial \vartheta(x,\tau)}{\partial \tau} = a\,\frac{\partial^2 \vartheta(x,\tau)}{\partial x^2},$$

$$\lambda \left|\frac{\partial \vartheta}{\partial x}\right|_{x=0} = -q(\tau),$$

$$\vartheta(x,0) = 0,\ \vartheta(0,\tau) = \vartheta_0,$$

$$\vartheta(\infty,\tau) = 0. \qquad (3)$$

The solution has the form

$$\vartheta(x,\tau) = \vartheta_0 \operatorname{erfc} \frac{x}{2\sqrt{a\tau}}. \qquad (4)$$

The law of variation with time of the power of a plane source which achieves the condition $\vartheta(0,\tau) = \vartheta_0$ is

$$q(\tau) = -\lambda \left.\frac{\partial \vartheta}{\partial x}\right|_{x=0} = -\frac{\lambda}{\sqrt{a}}\,\frac{\vartheta_0}{\sqrt{\pi\tau}} = -b\,\frac{\vartheta_0}{\sqrt{\pi\tau}}. \qquad (5)$$

Equation (1) may be written as

$$\frac{C_h}{F}\,\frac{d\vartheta_h}{d\tau} + 2b\,\frac{\vartheta_0}{\sqrt{\pi\tau}} = \frac{I_h^2 R_h}{F}. \qquad (6)$$

When the heater heat capacity C_h can be neglected, the formula for the thermal activity coefficient has the form

$$b = \frac{R_h\sqrt{\pi}}{2\vartheta_0 F}\, I_h^2 \sqrt{\tau}. \qquad (7)$$

Thus, by controlling the current in the heater circuit, we may determine the thermal activity coefficient of the test material. In order to examine the influence of heater thermal inertia, we shall consider the fact that in the measuring process the resistance the resistance only hypothetically remains constant, equal to R_{h0}, but in fact varies, albeit only slightly.

Substituting (2) into (6), we obtain

$$\frac{C_h}{F}\,\frac{d\vartheta_h}{d\tau} + 2b\,\frac{\vartheta_0}{\sqrt{\pi\tau}} = I_h^2\,\frac{R_{h0}}{F}\,[1 + \beta_{h0}(\vartheta_h - \vartheta_{h0})]. \qquad (8)$$

We express ϑ_h in terms of I_h. We may, with a high degree of accuracy, express the current I_h in terms of the increment of voltage supplied to the bridge and the heater resistance:

$$I_h = \frac{U_{b0}}{R_{h0}+R} + \frac{1}{R_{h0}+R}\,\Delta U_b - \frac{I_{h0}}{R_{h0}+R}\,\Delta R_h. \qquad (9)$$

The bridge imbalance ΔU is connected with the increment of voltage supplied to the bridge through the relation

$$\Delta U_b = -G\,\Delta U. \qquad (10)$$

We express ΔU in terms of the increment ΔR_h of heater resistance

$$\Delta U = I_{h0}\frac{R}{R_{h0}+R}\,\Delta R_h,\ R_1 = R_{h0}. \qquad (11)$$

Here $I_{h0} = U_{h0}/R_{h0} + R$, where U_{h0} is the voltage applied to the bridge.

Using (2), (10) and (11), we may write (9) as follows:

$$I_h = I_{h0} - \beta_{h0}\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right)(\vartheta_h - \vartheta_{h0}). \qquad (12)$$

Substituting (12) into (8), we obtain

$$\left(C_h \Big/ F\beta_{h0}\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right)\right)\frac{dI_h}{d\tau} + I_h^2\frac{R_{h0}}{F}\times$$
$$\times\left[1-(I_h - I_{h0})\Big/\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right)\right] = $$
$$= 2b\frac{\vartheta_{h0}}{\sqrt{\pi\tau}}. \qquad (13)$$

Let us evaluate each term of (13). It can be seen from (13), that, when $G = \infty$, the effect of the temperature dependence of the heater resistance and of its thermal inertia on heat transfer to the test body is not apparent. In reality $0 < G < \infty$, so that the coefficient in the derivative $dI_h/d\tau$, although small, is not zero. Experience with plane wire heaters [2] indicates that the value of the heater current will be 100–300 mA, while the difference $I_h - I_{h0} \approx 50 - 200$ mA.

Calculation shows that if the second term in square brackets in (13) is neglected, the error will be ≈0.3%.*

To evaluate the error introduced by calculations according to (7), we shall analyze the solution of the equation

$$\left(C_h \Big/ F\beta_{h0}\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right)\right)\frac{dI_h}{d\tau} +$$
$$+ I_h^2\frac{R_{h0}}{F} = 2b\frac{\vartheta_{h0}}{\sqrt{\pi\tau}}. \qquad (14)$$

Taking the initial conditions into account, the solution has the form

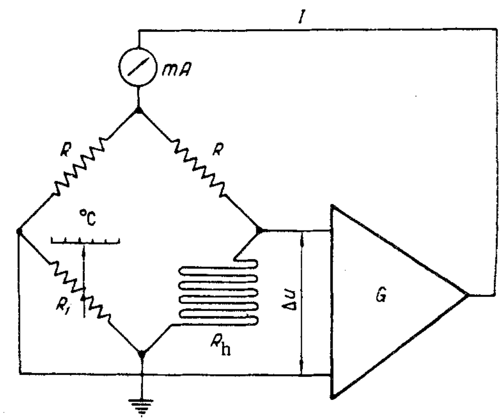
$$I_h = \sqrt{n/m}\,\tau^{-1/4}\,I_{-1/3}\left(\frac{4}{3}\tau^{3/4}\sqrt{mn}\right)\Big/$$
$$\Big/ I_{2/3}\left(\frac{4}{3}\tau^{3/4}\sqrt{mn}\right), \qquad (15)$$

where

$$m = \frac{R_{h0}}{C_h}\beta_{h0}\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right),$$

$$n = b\frac{2\vartheta_{h0}}{\sqrt{\pi}}\frac{F}{C_h}\beta_{h0}\frac{U_{h0}}{R_{h0}+R}\left(G\frac{R}{R_{h0}+R}+1\right).$$

$I_{-1/3}$, $I_{2/3}$ are modified Bessel functions of the first kind of order 1/3 and 2/3.



Main measuring circuit.

Let us now compare (15) with (7) solved for I_h:

$$I_h^{(15)}/I_h^{(7)} = I_{-1/3}\left(\frac{4}{3}\tau^{3/4}\sqrt{mn}\right)\Big/ I_{2/3}\left(\frac{4}{3}\tau^{3/4}\sqrt{mn}\right).$$

Direct calculation shows that

$$\frac{I_h^{(15)}}{I_h^{(7)}} = \frac{I_{-1/3}(1)}{I_{2/3}(1)} \approx 1.64,\quad \frac{I_h^{(15)}}{I_h^{(7)}} = \frac{I_{-1/3}(4)}{I_{2/3}(4)} \approx 1.04,$$

$$\frac{I_h^{(15)}}{I_h^{(7)}} = \frac{I_{-1/3}(10)}{I_{2/3}(10)} \approx 1.015.$$

Since in practice $\sqrt{mn} \approx 10^{(3-5)}$, even at quite small times of order 0.1–1 sec, calculations based on (7) will differ by less than 0.1% from those based on (15).

The accuracy of determination of the thermal activity may be increased if we consider that the eventual heater temperature depends on the gain of the amplifier. We shall calculate this temperature. For this purpose we express the current I_h in (8) in terms of temperature ϑ_h and put $\tau = \infty$ (steady conditions). We then obtain

$$\vartheta_h = \vartheta_{h0} - 1/\beta_{h0}\left(G\frac{R}{R_{h0}+R} - 1\right) = \vartheta_{h\infty}.$$

*Calculations were made for the following circuit parameter values: $R = R_h$, I_{h0} = 300 mA. $G = 1000$.

It is evident that the greater G and β_{h0}, the less will $\vartheta_{h\infty}$ differ from ϑ_{h0}.

For a more accurate determination of the thermal activity, $\vartheta_{h\infty}$ should be substituted for ϑ_{h0} in (7),

$$b = \frac{R_{h0}\sqrt{\pi}}{2\vartheta_{h\infty}F} I_h^2 \sqrt{\tau}.$$

NOTATION

λ) thermal conductivity; a) thermal diffusivity; U_{h0}) voltage drop at heater with bridge balanced.

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